Department of Economics **University of Manitoba**

ECON 7010: Econometrics I
FINAL EXAM, Dec. 19th, 2012

Instructor: **Ryan Godwin**
Instructions: **Answer ALL QUESTIONS, and put all answers in the booklet provided**
Time Allowed: **3 hours.**
Number of Pages: **6**

There are a total of 110 marks.

For PART A and PART B below, choose to answer *6 out of 7* of the following questions.

PART A: *State whether each of the following is TRUE or FALSE, and BRIEFLY explain your answer. Each question is worth 5 marks.*

Q.1) In the presence of heteroskedasticity the OLS estimator of the coefficient vector is inefficient and this causes the usual t tests and F tests to be invalid.

Q.2) The Hausman and Wu tests are designed to test if the instruments that we plan to use meet the requirement of being uncorrelated with the errors (at least asymptotically). In each case, a rejection of the null hypothesis would lead us to use OLS instead of I.V. estimation.

PART B: *Short answer. Each question is worth 5 marks.*

Q.3) Briefly describe how we could compare the efficiency of two estimators.

Q.4) Describe or illustrate one way in which you could test for heteroskedasticity.

Q.5) Describe the intuition behind the likelihood ratio test.

Q.6) Suppose that we wish to estimate a regression model by least squares, but this model is non-linear in the parameters. Explain why we will (generally) need to use a numerical algorithm to obtain the parameter estimates.

Q.7) In the context of OLS, where all usual assumptions are satisfied, is it more dangerous to include an irrelevant regressor, or exclude an irrelevant one?

PART C: *Long answer. Choose to answer* ***<u>2 out of 3</u>*** *of the following questions.*

Q.8) [10 marks]

Suppose that we have a linear multiple regression model,

$$y = X\beta + \varepsilon$$

$$= X_1\beta_1 + X_2\beta_2 + \varepsilon$$

where all of the usual assumptions are satisfied, except that $E(\varepsilon) = X_1\gamma$. That is, the mean vector for the disturbances is a linear combination of a subset of the regressors.

Let b_1 and b_2 be the OLS estimators of β_1 and β_2. Obtain the expressions for $E(b_1)$ and $E(b_2)$, and interpret your results.

Q.9) [10 marks]

Suppose we have a simple regression model, where the only regressor is a time-trend variable, that is: $x_t = 1,2,3, \dots, n$. So, the model is:

$$y_t = \beta_1 + \beta_2 x_t + \varepsilon_t \quad ; \quad \varepsilon_t \sim (0, \sigma^2)$$

Given the zero mean for the errors, and the non-random regressor, the OLS estimators (b_1 and b_2) of β_1 and β_2 are unbiased. It can be shown that

$$var(b_1) = \frac{2\sigma^2(n+1)(2n+1)}{n(n-1)(n+1)}$$

and

$$var(b_2) = \frac{12\sigma^2}{n(n-1)(n+1)}$$

(a) Are b_1 and b_2 mean-square consistent? Show any work.

(b) Are b_1 and b_2 weakly consistent? Explain.

(c) What does this imply about the usual assumption used to establish weak consistency of the OLS estimator for the regression coefficient vector?

Q.10) [10 marks]

Consider the linear multiple regression model, with *k* non-random regressors:

$$y = X\beta + \varepsilon \;; \qquad \varepsilon \sim N[0, \sigma^2 I] .$$

Suppose that one of the regressors is a dummy variable, and that this dummy is zero for all but one of the 'n' observations in the sample.

Prove that the OLS estimators for the coefficients of the other regressors in the model are the same as would be obtained if the dummy variable were omitted from the model, and the one observation for which the dummy is non-zero was also dropped from the sample.

[*Hint*: *OLS does not depend on the order of the data, so make things easier for yourself by assuming that the single 'special' observation is the last one in the sample*.]

PART D: Answer all questions.

Q.11) [20 marks]

Suppose that the true data-generating process is

$$y = X_1\beta_1 + X_2\beta_2 + \varepsilon \quad ; \quad \varepsilon \sim N(0, \sigma^2\Omega) \tag{1}$$

where Ω is a *known* matrix.

However, the model that we estimate by mistake, using *Generalized Least Squares*, is

$$y = X_1\beta_1 + \nu. \tag{2}$$

(a) Prove that the GLS estimator of β_1 will be biased, unless $(X_1'\Omega^{-1}X_2) = 0$.

[6 marks]

(b) How do you think the variability of this estimator will compare with the variability of the estimator of β_1 when the GLS estimator is applied to equation (1)?

[4 marks]

(c) Now let's suppose that the true data-generating process is given by

$$y = X_1\beta_1 + \varepsilon \quad ; \quad \varepsilon \sim N(0, \sigma^2\Omega) \tag{3}$$

But the model we estimate by GLS is

$$y = X_1\beta_1 + X_2\beta_2 + u. \tag{4}$$

Prove that the GLS estimators of both β_1 and β_2 are unbiased in this case.

[**Hint**: Note that $X_1 = (X_1 \quad X_2)\begin{pmatrix} I \\ 0 \end{pmatrix} = XS$, say.]

[6 marks]

(d) What other properties would you expect these last GLS estimators of β_1 and β_2 to possess? (No formal proofs are needed).

[4 marks]

Q.12) [20 marks, each part worth 4 marks]

A discrete random variable, $= 0, 1, 2, 3, \ldots$, is said to have a Poisson distribution with parameter $\lambda > 0$, if the probability mass function of y is given by:

$$p(y \mid \lambda) = \frac{\lambda^y}{e^\lambda y!}$$

The mean and variance of y are both equal to λ.

(a) Assuming independence, write down the likelihood function $L(\lambda \mid y_1, y_2, \ldots, y_n)$

(b) Derive the maximum likelihood estimator for λ in this model.

(c) Prove that this estimator is unbiased.

Recall that Fisher's information matrix is $I(\lambda) = -E[H(\lambda)]$, where $H(\lambda)$ is scalar, and is the second derivative of the log-likelihood function.

(d) Derive the variance of the MLE for λ. How can you obtain an estimator for this variance? What property of MLEs are you using?

(e) What asymptotic properties does $\tilde{\lambda}$ have?

Q.13) [20 marks, each part worth 5 marks]

In 1955, Simon Kuznets hypothesized that as economic development occurs, income inequality first increases, then after some ‘turning point’ it decreases. That is, he hypothesized an ‘inverted U-curve’. This hypothesis has been quite controversial, and many authors have found the reverse result – a ‘U curve’, rather than an ‘inverted U-curve’. The following EViews output is based on a sample of 32 cross-section data values, for different countries, and the variables are:

GINI	Gini coefficient for income inequality – ***the dependent variable in the model***
RY	*Per capita* GDP, in real international dollars
DUM	Dummy variable (= 1 for developing countries; = 0 otherwise)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.415418	0.045531	9.123762	0.0000
RY	4.98E-05	3.79E-05	1.312220	0.2001
RY^2	-5.52E-09	5.36E-09	-1.030146	0.3118
DUM	-0.167568	0.088942	-1.884022	0.0700

R-squared		Mean dependent var	0.437969
Adjusted R-squared	0.185091	S.D. dependent var	0.117046
S.E. of regression	0.105660	Akaike info criterion	-1.540709
Sum squared resid		Schwarz criterion	-1.357492

(a) A friend says: “On the basis of these results, I’d just consider a linear relationship between GINI and RY”. Do you agree? Justify your answer by means of formal hypothesis testing, ***stating any assumptions that you use.***

(b) Another friend says: “The relationship between GINI and real GDP is the same for developing countries as for developed countries.” Do you agree? Again, justify your answer by means of formal hypothesis testing, ***stating any assumptions that you use.***

(c) I forgot to report the value for R^2 and the “sum of squared residuals”. Calculate both of these values. Explain exactly how the value of R^2 should be interpreted here.

(d) Construct a 95% confidence interval for the coefficient of RY, and carefully interpret its meaning.